Results obtained and future plans

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Fields of interests and results obtained

My active scientific carrier began in the early nineties after getting admission as a Ph.D. student in 1990.

As a graduate student, I made first steps in the world of theoretical physics studying the **Einstein - de Broglie particle-soliton concept** and further applying this concept to simulate stationary states of an electron in a hydrogen atom.

In 1991 I became interested in **gravitation and cosmology** which resulted in intensive study of **soliton-like configurations** in general relativity and evolution of nonlinear spinor field in anisotropic back-ground. I got my Ph.D. in theoretical physics in 1993.

Fields of interests and results obtained

In 1994 when I moved to Joint Institute for Nuclear Research (JINR), Dubna, I refocused my interest to electrodynamics with toroid polarization.

In 1998 I got involved in some projects where as a starting point inherent symmetry of the manifold has been expounded.

Though the fields of my interest are scattered, there is a common string - it is the nonlinearity. Nonlinear differential equations and group theory are also among the fields of my interest.

Fields of interests and results obtained

Here is a short account of the fields of my interest and results obtained.

It should be emphasized that my Ph.D. thesis Multidimensional solitons in nonlinear models with gravitation, that I defended in 1993 was based on soliton in general relativity, and quantum mechanics with extended particles, though some works on anisotropic cosmology were also included.

Since 2001 I have been completely involved in anisotropic cosmological models. My D.Sc. thesis Spinor field in anisotropic cosmology, that I defended in 2009 was completely based on my results obtained in cosmology. Recently I am engaged in string cosmological models and loop quantum cosmology.

Spinor field in cosmology

In 1991 when I made my first steps in cosmology, I wanted to encounter two following questions:

(i) Expanding model of the Universe leads to initial singularity. Is it an inherent property of relativistic cosmological models or it is only a consequence of the specific simplifying assumptions underlying these models? Is it possible to obtain a singularity-free cosmological solution?

(ii) The present day Universe is surprisingly isotropic and homogeneous. Was the picture same always? What if we begin with a more realistic model that is initially anisotropic? Will there be an asymptotically isotropization process so that the initially anisotropic Universe evolves into an isotropic one?

Spinor field in cosmology

Most of the cosmological models those days were based on perfect fluids or scalar field. Our goal was to study the role of spinor fields in answering the foregoing questions.

Nonlinear self-couplings of spinor field may arise as a consequence of geometrical structure of the spacetime and more precisely, because of the existence of torsion.

Being related to almost all stable elementary particles such as proton, electron and neutrino, spinor field, especially Dirac spin-$1/2$ play a principal role at the microlevel. However, in cosmology, the role of spinor field was generally considered to be restricted.

Spinor field in cosmology

So we introduced a nonlinear spinor field into the Einstein system of equation

$$L_{\rm sp} = \frac{\imath}{2}\left[\bar{\psi}\gamma^{\mu}\nabla_{\mu}\psi - \nabla_{\mu}\bar{\psi}\gamma^{\mu}\psi\right] - m_{\rm sp}\bar{\psi}\psi - F, \tag{1}$$

where the nonlinear term F describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field: $F = F(K)$, with K, taking one of the following expressions: $\{I,\ J,\ I+J,\ I-J\}$.

Assuming that $\psi = \psi(t)$ for a power law nonlinearity

$$F(K) = \sum_{j} \lambda_j K^{n_j}, \tag{2}$$

Spinor field in cosmology

it was found that for a suitable choice of problem parameter the spinor field can

(i) accelerate the isotropization process, i.e., compared to the scalar field or perfect fluid the isotropization process of the initially anisotropic spacetime takes place in an earlier time;

(ii) give rise to a singularity-free Universe. In this case it is possible to obtain periodic solution that begins at some non-singular state, expands to some maximum value and then decreases until some minimum but non-zero value only to expand again.

(iii) generate late time acceleration. In this case can be considered as an alternative description of dark energy.

These findings lead many researchers to consider spinor field in an alternative source to answer many unanswered questions in cosmology.

Given the role that spinor field can play in the evolution of the Universe, question that naturally pops up is, if the spinor field can redraw the picture of evolution caused by perfect fluid and dark energy, **is it possible to simulate perfect fluid and dark energy by means of a spinor field?**

And soon it was found that the spinor field can **simulate the different characteristics of matter such as perfect fluid obeying barotropic equation of state, quintessence, phantom matter, chapligyn gas, cosmic string, Van der Waals gas, modified quintessence etc.**

Spinor field in cosmology

But those results were obtained assuming that the spinor field possesses only **diagonal components of the energy-momentum tensor (EMT).**

But some recent works show that depending on the spacetime geometry the spinor field may posses **non-zero non-diagonal components of EMT.** And this very fact plays a crucial role on the character of both **spacetime geometry and spinor field itself.**

In particular, it was found that **in case of a full Bianchi type-I spacetime, the non-diagonal components of EMT lead to the elimination of the bilinear spinor forms which result in vanishing massive term and nonlinear term in the spinor field Lagrangian.**

Spinor field in cosmology and observation

Our works in cosmology are mainly theoretical where we simulate different types of picture of evolution of the Universe by manipulating problem parameters such as spinor mass, power of nonlinearity etc.

To find the numerical values of problem parameters, that gives a realistic picture of evolution, lately we are comparing the theoretical solutions to **astrophysical observations** exploiting the **maximum likelihood method.**

Using this method we have found the age of the Universe (T_U). (i) in case of the soft beginning of expansion (initial speed of expansion in a point of singularity is equal to zero) we have T_U **= 15 billion years,** whereas in case of the hard beginning (nontrivial initial speed) we found T_U **= 13.7 billion years.**

Isotropic and Anisotropic Dark Energy Models

Detection and further experimental reconfirmation of current **cosmic acceleration** pose to cosmology a fundamental task of identifying and revealing the cause of such phenomenon.

On the other hand theoretical arguments and experimental data suggest that the Universe might have an **anisotropic phase of evolution in the remote past.**

Motivated by those facts we have studied the evolution of an initially anisotropic Universe given by different Bianchi type cosmological models filled with different types of dark energy, fluids with variable equation od state (EoS) parameters, respectively. Corresponding field equations were solved and the results were compared with the experimental data.

Isotropic and Anisotropic Dark Energy Models

A new class of dark energy models in anisotropic space-time with time dependent equation of state (EoS) parameter

$$\omega = \frac{p}{\varepsilon}, \tag{3}$$

where p is the pressure and ε is the energy density of dark energy, and constant deceleration parameter has been investigated.

The Einstein's field equations have been solved by applying a variation law for generalized Hubble's parameter, which generates power-law type and exponential type solutions.

Isotropic and Anisotropic Dark Energy Models

The existing range of the dark energy EoS parameter for both models is found to be in good agreement with the three recent observations:

(i) SNe Ia data (2003);

(ii) SNe Ia data collaborated with CMBR anisotropy and galaxy clustering statistics (2004);

(iii) a combination of cosmological datasets coming from CMB anisotropies, luminosity distances of high redshift type Ia supernovae and galaxy clustering (2009).

Isotropic and Anisotropic Dark Energy Models

The cosmological constant Λ is found to be a decreasing function of time and it approaches a small positive value at the present epoch which is corroborated by results from recent supernovae Ia observations.

It has also been suggested that the dark energy that explains the observed accelerating universe may arise due to the contribution to the vacuum energy of the EoS in a time dependent background.

Geometric and kinematic properties of the models and the behavior of the anisotropy of the dark energy have been carried out. Analogical study has been carried out within the scope of Bianchi type VI (BVI), BVI_0, BV, $BIII$, BI and FRW models.

string cosmological model in loop quantum cosmology

The loop quantum cosmology of the Bianchi type I and II string cosmological model in the presence of a homogeneous magnetic field is studied.

The effective equations which provide modifications to the classical equations of motion due to quantum effects are presented.

The numerical simulations confirm that the big bang singularity is resolved by quantum gravity effects.

A parallel between the classical and quantum approaches is drawn.

Potential for Isotropization and Inflation

Scalar field plays a very important role in cosmology. Due to the fact that the scalar field possesses zero spin, it was basically considered in isotropic cosmological models. If considered in an anisotropic model, the linear scalar field does not lead to isotropization of expansion process. One needs to introduce scalar field with nonlinear potential for the isotropization process to take place.

General form of scalar field potentials leading to the asymptotic isotropization in case of Bianchi type-I cosmological model, and inflationary regime in case of isotropic space-time is obtained. In doing both direct and inverse problems were solved, where by direct problem we mean to find metric functions and scalar field for the given potential, whereas, the inverse problem means to find the potential and scalar field for the given metric function.

Role of Λ term

The role of a Λ term in the evolution of the Universe is thoroughly studied. It was shown that depending on the sign the Λ term gives three different type of evolution scenario, namely

(i) a positive Λ leads to the accelerating mode of the Universe that never halts;
(ii) if Λ is trivial, at some stage the evolution of the Universe comes stand-still;
(iii) a negative Λ results in the cyclic mode of expansion of the Universe.

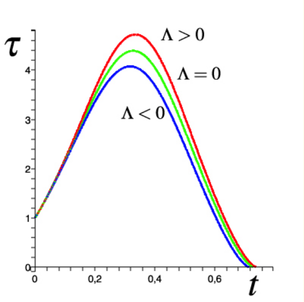
It should be noted that the Λ term plays crucial role in the evolution only in absence of a viscous fluid.

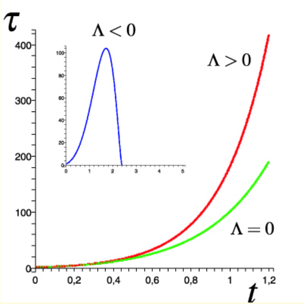
Role of a viscous fluid and Van der Waals gas

The role of a viscous fluid in the evolution of the Universe is thoroughly studied. It was shown that depending on the viscosity the Universe may allow expanding or cyclic mode of evolution independent to the sign of Λ. It was also shown that together with nonlinear spinor field the viscous fluid gives rise to a Big Rip type solution when both the Universe and the energy density becomes infinite within the finite range of time.

It was shown that the corresponding system allows exact solutions only under some restrictions of viscosity. The complete qualitative analysis of the system was carried out.

Role of a viscous fluid



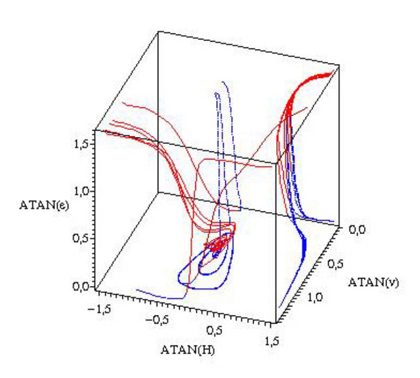


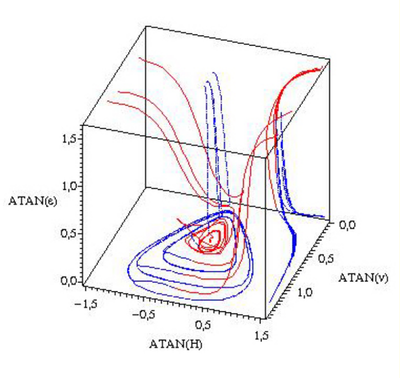
Evolution of the Universe for $\kappa < 1$.

Evolution of the Universe for $\kappa > 1$.

Evolution of the Universe when the bulk viscosity is proportional to expansion and the shear viscosity is constant.

Role of a viscous fluid

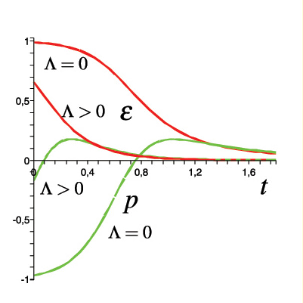


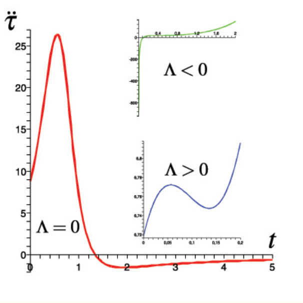


Trajectory of evolution in case of a viscous fluid and a nonlinear spinor field.

Trajectory of evolution in case of a viscous fluid and an interacting system of nonlinear spinor field and scalar fields.

It is shown that the Van der Waals gas gives rise to two phases of evolution, the first one is the inflationary phase, when the Universe expands exponentially. This phase smoothly transfers to the deceleration phase.





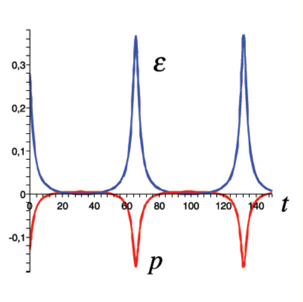
Evolution of energy density ε and pressure p.

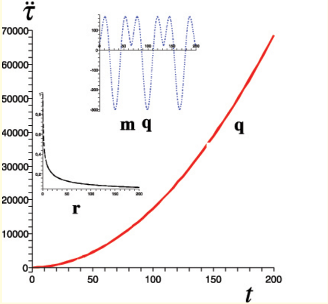
View of acceleration in presence of a Van der Waals gas.

Quintessence with modified EoS

In order to avoid the problem of eternal acceleration quintessence with a modified EoS was proposed:

$$p_{\rm mq} = -w(\varepsilon_{\rm mq} - \varepsilon_{\rm cr}), \quad w \in [0,\, 1), \quad \varepsilon_{\rm cr} - \text{some critical density}$$





Evolution of energy density and pressure.

View of acceleration for different sources.

Spinor fields in static plane-symmetric space-time

We study the role of a nonlinear spinor field within the scope of static plane-symmetric space-time

$$ds^2 = e^{2\chi} dt^2 - e^{2\alpha} dx^2 - e^{2\beta}(dy^2 + dz^2), \quad (4)$$

with χ, α, β being the function of x only, obeying

$$\alpha = 2\beta + \chi. \quad (5)$$

It was shown that
(i) The energy density and total energy of the system is limited;
(ii) introduction of spinor field nonlinearity on account of its proper gravitational field generates field configurations with finite energy density, and limited total energy, spin and charge.

Bel-Robinson tensor

The lack of a well-posed definition of local energy-momentum tensor is the consequence of the Principle of Equivalence, which lies at the heart of Einstein's theory of general relativity. Nevertheless, quest for the local tensors describing the strength of gravitational field has long been going on.

One of the first successful attempt to address this problem was taken by Bel and independently Robinson, where in analogy with the electromagnetic energy-momentum tensor, they constructed a four-index tensor (BR) for the gravitational field in vacuum.

Bel-Robinson tensor

Within the scope of BI cosmological model three alternative definitions of BR tensor are considered:

It is shown that the definition used by Deser *et. al.* is consistent with the Einstein equations for empty space.

The second definition used by Teyssandier is free from this restriction, but BR defined in this way is not totally symmetric.

Definition used by Senovilla and Bergqvist has all the symmetries and the dominant super energy property (DSEP) is satisfied.

Quantum Mechanics - Extended Particle Formalism

In the framework of his theory of double solution, Louis de Broglie made an attempt to represent the electron as a source of waves obeying the Schroedinger equation.

Later he modified his model showing that the electron should be described by regular solutions to some nonlinear equation coinciding with the Schroedinger one in the linear approximation. This scheme became famous as a causal nonlinear interpretation of quantum mechanics.

Developing this concept, de Broglie remarked that it has much in common with Einstein's ideas about unified field theory.

Quantum Mechanics - Extended Particle Formalism

The Einstein - de Broglie particle-soliton concept is applied to simulate stationary states of an electron in a hydrogen atom, where the electron is described by the localized regular solutions to some nonlinear equations.

In the framework of Synge model for interacting scalar and electromagnetic fields a system of integral equations has been obtained.

The asymptotic expressions for physical fields, describing soliton moving around the fixed Coulomb center, have been obtained with the help of integral equations.

Quantum Mechanics - Extended Particle Formalism

It is shown that the electron-soliton center travels along some stationary orbit around the Coulomb center. The electromagnetic radiation is absent as the Poynting vector has non-wave asymptote $O(r^{-3})$ after averaging over angles, i.e. the existence of spherical surface corresponding to null Poynting vector stream, has been proved.

Vector lines for Poynting vector are constructed in asymptotic area. Moreover, some first principles that could serve as the foundation for quantum theory of extended particles are formulated.

Solitons in General Relativity

Solitons with spherical and/or cylindrical symmetry to the equations governing the interacting system of scalar, electromagnetic, and gravitational fields have been obtained.

As a particular case it is shown that the equations of motion admit a special kind of solution with a sharp boundary, known as droplets. For these solutions, the physical fields vanish and the space-time is flat outside of the critical sphere or cylinder. Therefore, the mass and the electric charge of these configurations are zero.

It is noteworthy that the effective potentials in this case possess confining property, i.e., create a strong repulsion on certain surfaces in configuration space.

Solitons in General Relativity

Scalar fields with induced non-linearity has been studied in external FRW and Gödel Universes.

It has been shown that in FRW Universe with $k = +1$ all the solutions are localized in the region $0 \leq r \leq 1$.

Beside the droplets few other special field configurations (anti-droplets, hats) have been obtained and their stability has been studied in details.

Symmetry methods in physics

The gauge symmetry inherent in the concept of manifold has been discussed. Within the scope of this symmetry the linear connection or displacement field can be considered as a natural gauge field on the manifold.

The gauge invariant equations for the displacement field have been derived. It has been shown that the energy-momentum tensor of this field conserves and hence the displacement field can be treated as one that transports energy and gravitates.

To show the existence of the solutions of the field equations we have derived the general form of the displacement field in Minkowski space-time which is invariant under rotation and space and time inversion. With this anzats we found spherically-symmetric solutions.

Electrodynamics with toroid polarizations

With regard to the toroid contributions, a modified system of equations of electrodynamics moving continuous media has been obtained. Alternative formalisms (Lagrangian, Hamiltonian and Gauge-like one) to introduce the toroid moment contributions in the equations of electromagnetism has been worked out.

The two four-potential formalism has been further developed for the equations obtained. It has been shown that the modified system is Lorentz covariant. Lorentz transformation laws for the toroid polarizations has been given. Covariant form of equations of electrodynamics of continuous media with toroid polarizations has been written.

Anisotropic string cosmology

Within the scope of BI and BVI cosmological models the evolution of the Universe in presence of a cosmic string and magnetic flux are studied. In case of BI space-time the system can be solved only for some special cases.

In case of BVI, the presence of a magnetic field imposes severe restrictions regarding the consistency of the field equations. These difficulties could be overtaken working either in a BVI_0 space-time or assuming a particular coordinate-dependence of the magnetic field. Using a few plausible assumptions regarding the parametrization of the cosmic strings, some exact analytical solutions are presented. Their asymptotic behavior for large time is exhibited.

Future plans

Though a lot has been done still there is a lot of scope to develop cosmological models with spinor field. Especially the recent results dictate that we should consider some other forms of spinor fields.

One of the possibilities to consider 8-components spinor field what I plan to do in near future.

Another aspect of my study will be to consider the spinor field with the Lagrangian that gives spinor field equation of higher order:

$$L_{\rm sp} = \frac{\imath}{2}\nabla_\mu \bar\psi \left(\gamma^\nu j_\nu\right)\nabla^\mu \psi - m_{\rm sp}\bar\psi\psi - F, \quad j_\nu = \bar\psi\gamma_\nu\psi. \tag{6}$$

Future plans

Given the quantum character of spinor field I plan to study the anisotropic spinor cosmology from quantum point of view.

The discovery of accelerated mode of expansion of the Universe has opened new horizons of cosmology. A number of models have been proposed, but the mystery behind this acceleration is far from over. Taking into account the observational data I want to continue my study with dark energy and dark matter.

I also plan to study different aspects of Loop Quantum Cosmology within the scope of anisotropic cosmological models.

Future plans

Another way to face the challenges of modern cosmology is to modify the standard Einstein theory of gravity. I plan to consider cosmological models within the scope of modified gravity such as $f(R)$ theory, $f(R, T)$ theory, etc.

With the development of technology today we know more about the cosmos than ever before. Now the sky has opened up all its treasure so that we could understand and explain the the entire history of the Universe. Taking this in mind I plan to compare the theoretical results of spinor cosmological models with the available experimental data to fix the problem parameters that gives the best matching with the real picture of the Universe.